Cleeve Prior CE Primary School

Maths progression document including Calculations policy 2026-27

Cleeve Prior
CE PRIMARY SCHOOL

This document shows the expected progression in Maths knowledge and skills across our Primary school.

The calculation policy is taken from the White Rose Calculation policy and is a guide to progression in calculations. It is refined to show the strategies we use at Cleeve Prior in our mixed-aged classes when teaching calculations.

It is designed to show the development of skills in calculations using concrete, pictorial and abstract concepts.

Its aim is to help all learner become more efficient and effective at carrying out increasingly complex calculations.

Cleeve Prior CE Primary School: Maths Knowledge and Skills Progression Document

KNOWLEDGE								
	Nursery	**Reception**	**Year 1**	**Year 2**	**Year 3**	**Year 4**	**Year 5**	**Year 6**
Place Value	Take part in finger rhymes with numbers Recite numbers past 5 Count in every day contexts Say one number for each item in order Develop fast recognition of up to 3 objects Experiment with their own symbols and marks as well as numerals	Have a deep understanding of numbers to 10, including the composition of each number Subitise up to 5 Verbally count beyond 20, recognising the pattern of the number system Compare quantities up to 10 in different contexts, recognising when one is greater than, less than or the same as the other quantity Explore and represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be distributed evenly	count to and across 100, forwards and backwards, beginning with 0 or 1, or from any given number Count numbers to 100 in numerals; count in multiples of twos, fives and tens identify and represent numbers using objects and pictorial representations read and write numbers to 100 in numerals read and write numbers from 1 to 20 in numerals and words given a number, identify one more and one less	count in steps of 2, 3, and 5 from 0, and in tens from any number, forward and backward read and write numbers to at least 100 in numerals and in words identify, represent and estimate numbers using different representations, including the number line recognise the place value of each digit in a two-digit number (tens, ones) compare and order numbers from 0 up to 100; use and = signs	count from 0 in multiples of 4, 8, 50 and 100; find 10 or 100 more or less than a given number identify, represent and estimate numbers using different representations read and write numbers up to 1000 in numerals and in words recognise the place value of each digit in a three-digit number (hundreds, tens, ones) compare and order numbers up to 1000	count in multiples of 6, 7, 9, 25 and 1000 count backwards through zero to include negative numbers identify, represent and estimate numbers using different representations read Roman numerals to 100 (I to C) and know that over time, the numeral system changed to include the concept of zero and place value find 1000 more or less than a given number recognise the place value of each digit in a four-digit number (thousands, hundreds, tens, and ones) order and compare numbers beyond 1000	count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000 count forwards and backwards with positive and negative whole numbers, including through zero Read, write, order and compare numbers to at least 1 000 000 and determine the value of each digit Read Roman numerals to 1000 (M) and recognise years written in Roman numerals Read, write, order and compare numbers to at least 1 000 000 and determine the value of each digit	Read, write, order and compare numbers up to 10 000 000 and determine the value of each digit Read, write, order and compare numbers up to 10 000 000 and determine the value of each digit

Addition and Subtraction	React to changes of an amount in a group Compare amounts by saying 'lots' 'more' or 'the same'		add and subtract one-digit and two-digit numbers to 20, including zero	add and subtract numbers using concrete objects, pictorial representations, and mentally, including: a two-digit number and ones; a two-digit number and tens; two two-digit numbers; adding three one-digit numbers	add and subtract numbers mentally, including: a three-digit number and ones; a three-digit number and tens; a three-digit number and hundreds add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction	add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate	add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction) add and subtract numbers mentally with increasingly large numbers	perform mental calculations, including with mixed operations and large numbers use their knowledge of the order of operations to carry out calculations involving the four operations
Multiplication and Division				recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot	recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables	recall multiplication and division facts for multiplication tables up to 12 × 12 use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers recognise and use factor pairs and commutativity in mental calculations	identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers know and use the vocabulary of prime numbers, prime factors and composite (nonprime) numbers establish whether a number up to 100 is prime and recall prime numbers up to 19 recognise and use square numbers and cube numbers, and the notation for squared (2) and cubed (3)	identify common factors, common multiples and prime numbers use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy
Fractions, Decimals and Percentages			recognise, find and name a half as one of two equal parts of an object, shape or quantity	recognise, find, name and write fractions ½, 1/3, 1/4, and ¾ of a length, shape, set of objects or quantity	count up and down in tenths; recognise that tenths arise from dividing an object into 10 equal parts and in dividing one-digit	count up and down in hundredths; recognise that hundredths arise when dividing an object by one	identify, name and write equivalent fractions of a given fraction, represented visually, including	use common factors to simplify fractions; use common multiples to express fractions in the same denomination

			recognise, find and name a quarter as one of four equal parts of an object, shape or quantity	Recognise the equivalence of 2/4 and ½ write simple fractions	numbers or quantities by 10 recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators recognise and use fractions as numbers: unit fractions and non-unit fractions with small denominators recognise and show, using diagrams, equivalent fractions with small denominators compare and order unit fractions, and fractions with the same denominators add and subtract fractions with the same denominator within one whole	hundred and dividing tenths by ten. recognise and show, using diagrams, families of common equivalent fractions add and subtract fractions with the same denominator recognise and write decimal equivalents of any number of tenths or hundredths recognise and write decimal equivalents to ½, ¼, ¾ round decimals with one decimal place to the nearest whole number compare numbers with the same number of decimal places up to two decimal places	tenths and hundredths recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements compare and order fractions whose denominators are all multiples of the same number add and subtract fractions with the same denominator and denominators that are multiples of the same number multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams read and write decimal numbers as fractions recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents round decimals with two decimal places to the nearest whole number and to one decimal place	compare and order fractions, including fractions > 1 add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions multiply simple pairs of proper fractions, writing the answer in its simplest form divide proper fractions by whole numbers identify the value of each digit in numbers given to three decimal places associate a fraction with division and calculate decimal fraction equivalents [for example, 0.375] for a simple fraction recall and use equivalences between simple fractions, decimals and percentages, including in different contexts

							read, write, order and compare numbers with up to three decimal places recognise the per cent symbol (%) and understand that per cent relates to 'number of parts per hundred', and write percentages as a fraction with denominator 100, and as a decimal	
Measure	Compare sizes, weights etc using gesture and language 'bigger/smaller' 'high/low' 'tall' 'heavy'		compare, describe and solve practical problems for: lengths and heights; mass/weight; capacity and volume; time measure and begin to record the following: lengths and heights; mass/weight; capacity and volume; time (hours, minutes, seconds)	choose and use appropriate standard units to estimate and measure length/height in any direction (m/cm); mass (kg/g); temperature (°C); capacity (litres/ml) to the nearest appropriate unit, using rulers, scales, thermometers and measuring vessels compare and order lengths, mass, volume/capacity and record the results using >, < and =	measure, compare, add and subtract: lengths (m/cm/mm); mass (kg/g); volume/capacity (l/ml) measure the perimeter of simple 2-D shapes	Convert between different units of measure [for example, kilometre to metre; hour to minute] estimate, compare and calculate different measures measure and calculate the perimeter of a rectilinear figure (including squares) in centimetres and metres find the area of rectilinear shapes by counting squares	convert between different units of metric measure understand and use approximate equivalences between metric units and common imperial units such as inches, pounds and pints measure and calculate the perimeter of composite rectilinear shapes in centimetres and metres calculate and compare the area of rectangles (including squares) and including using standard units, square centimetres (cm2) and square metres (m2) and estimate the area of irregular shapes estimate volume [for example, using blocks to build cuboids] and	use, read, write and convert between standard units, converting measurements of length, mass, volume and time from a smaller unit of measure to a larger unit, and vice versa, using decimal notation to up to 3dp convert between miles and kilometres recognise that shapes with the same areas can have different perimeters and vice versa recognise when it is possible to use formulae for area and volume of shapes calculate the area of parallelograms and triangles calculate, estimate and compare volume of cubes and cuboids using

							capacity [for example, using water]	standard units, including cubic centimetres (cm3) and cubic metres (m3), and extending to other units
Money			recognise and know the value of different denominations of coins and notes	recognise and use symbols for pounds (£) and pence (p); combine amounts to make a particular value find different combinations of coins that equal the same amounts of money	add and subtract amounts of money to give change, using both £ and p in practical contexts	estimate, compare and calculate different measures, including money in pounds and pence		
Time			sequence events in chronological order using language [for example, before and after, next, first, today, yesterday, tomorrow, morning, afternoon and evening] recognise and use language relating to dates, including days of the week, weeks, months and years tell the time to the hour and half past the hour and draw the hands on a clock face to show these times	compare and sequence intervals of time tell and write the time to five minutes, including quarter past/to the hour and draw the hands on a clock face to show these times know the number of minutes in an hour and the number of hours in a day	tell and write the time from an analogue clock, including using Roman numerals from I to XII, and 12-hour and 24-hour clocks estimate and read time with increasing accuracy to the nearest minute; record and compare time in terms of seconds, minutes and hours; use vocabulary such as o'clock, a.m./p.m., morning, afternoon, noon and midnight know the number of seconds in a minute and the number of days in each month, year and leap year compare durations of events [for example to calculate the time	read, write and convert time between analogue and digital 12- and 24-hour clocks		use, read, write and convert between standard units, converting measurements of time from a smaller unit of measure to a larger unit, and vice versa

					taken by particular events or tasks]			
Geometry	Notice patterns and arrange things in patterns Talk about and identify patterns around them Understand position through words 'the bag is under the table' Talk about and explore 2D and 2D shapes Select shapes appropriately Describe a familiar route Extend and create ABAB patterns	Identify and name 2D shapes – circle, triangle, square, rectangle Use prepositional language 'in, on under, over, beside, between' etc	recognise and name common 2-D shapes [for example, rectangles (including squares), circles and triangles] recognise and name common 3-D shapes [for example, cuboids (including cubes), pyramids and spheres]	identify and describe the properties of 2-D shapes, including the number of sides and line symmetry in a vertical line identify 2-D shapes on the surface of 3-D shapes, [for example, a circle on a cylinder and a triangle on a pyramid] compare and sort common 2-D shapes and everyday objects recognise and name common 3-D shapes [for example, cuboids (including cubes), pyramids and spheres] compare and sort common 3-D shapes and everyday objects	draw 2-D shapes make 3-D shapes using modelling materials; recognise 3-D shapes in different orientations and describe them recognise angles as a property of shape or a description of a turn identify right angles, recognise that two right angles make a half-turn, three make three quarters of a turn and four a complete turn; identify whether angles are greater than or less than a right angle identify horizontal and vertical lines and pairs of perpendicular and parallel lines	compare and classify geometric shapes, including quadrilaterals and triangles, based on their properties and sizes identify lines of symmetry in 2-D shapes presented in different orientations identify acute and obtuse angles and compare and order angles up to two right angles by size identify lines of symmetry in 2-D shapes presented in different orientations complete a simple symmetric figure with respect to a specific line of symmetry	distinguish between regular and irregular polygons based on reasoning about equal sides and angles. use the properties of rectangles to deduce related facts and find missing lengths and angles identify 3-D shapes, including cubes and other cuboids, from 2-D representations know angles are measured in degrees: estimate and compare acute, obtuse and reflex angles draw given angles, and measure them in degrees identify: angles at a point and one whole turn (total 360°), angles at a point on a straight line and 1/2 a turn (total 180°) other multiples of 90°	draw 2-D shapes using given dimensions and angles compare and classify geometric shapes based on their properties and sizes illustrate and name parts of circles, including radius, diameter and circumference and know that the diameter is twice the radius recognise, describe and build simple 3-D shapes, including making nets find unknown angles in any triangles, quadrilaterals, and regular polygons recognise angles where they meet at a point, are on a straight line, or are vertically opposite, and find missing angles
Position and Direction			describe position, direction and movement, including whole, half, quarter and three-quarter turns	order and arrange combinations of mathematical objects in patterns and sequences use mathematical vocabulary to describe position, direction and movement, including movement in a straight line and distinguishing		describe positions on a 2-D grid as coordinates in the first quadrant describe movements between positions as translations of a given unit to the left/right and up/down	identify, describe and represent the position of a shape following a reflection or translation, using the appropriate language, and know that the shape has not changed	describe positions on the full coordinate grid (all four quadrants) draw and translate simple shapes on the coordinate plane, and reflect them in the axes

				between rotation as a turn and in terms of right angles for quarter, half and three-quarter turns (Clockwise and anticlockwise)		plot specified points and draw sides to complete a given polygon		
Statistics				interpret and construct simple pictograms, tally charts, block diagrams and simple tables	interpret and present data using bar charts, pictograms and tables	interpret and present discrete and continuous data using appropriate graphical methods, including bar charts and time graphs	complete, read and interpret information in tables, including timetables	interpret and construct pie charts and line graphs and use these to solve problems

SKILLS								
	Nursery	**Reception**	**Year 1**	**Year 2**	**Year 3**	**Year 4**	**Year 5**	**Year 6**
Place Value	Solve real world mathematical problems with numbers up to 5	Automatically recall number bonds to 5, and some number bonds to 10, including double facts Compare objects based on characteristics Make sets of objects based on characteristics		use place value and number facts to solve problems	solve number problems and practical problems involving these ideas	round any number to the nearest 10, 100 or 1000 solve number and practical problems that involve all of the above and with increasingly large positive numbers	interpret negative numbers in context round any number up to 1 000 000 to the nearest 10, 100, 1000, 10 000 and 100 000 solve number problems and practical problems that involve all of the above	round any whole number to a required degree of accuracy use negative numbers in context, and calculate intervals across zero solve number and practical problems that involve all of the above
Addition and Subtraction			solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as 7 =? - 9	solve problems with addition and subtraction: using concrete objects and pictorial representations, including those involving numbers, quantities and measures; applying their increasing knowledge of mental and written methods	solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction	solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why	solve addition and subtraction multistep problems in contexts, deciding which operations and methods to use and why solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the	solve addition and subtraction multistep problems in contexts, deciding which operations and methods to use and why

							meaning of the equals sign	
Multiplication and Division			solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher	calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts	write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects	multiply two-digit and three-digit numbers by a one-digit number using formal written layout solve problems involving multiplying and adding, including using the distributive law to multiply two-digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects	multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers multiply and divide numbers mentally drawing upon known facts divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context multiply and divide whole numbers and those involving decimals by 10, 100 and 1000 solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes solve problems involving multiplication and	multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context perform mental calculations, including with mixed operations and large numbers solve problems involving addition, subtraction, multiplication and division

							division, including scaling by simple fractions and problems involving simple rates solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign	use their knowledge of the order of operations to carry out calculations involving the four operations
Fractions, Decimals and Percentages					solve problems that involve all of the above	solve problems involving increasingly harder fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions where the answer is a whole number solve simple measure and money problems involving fractions and decimals to two decimal places	solve problems which require knowing percentage and decimal equivalents of ½, ¼, 1/5, 2/5, 4/5 and those fractions with a denominator of a multiple of 10 or 25	
Ratio and Proportion			solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as 7 =? – 9	recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems	solve problems, including missing number problems			use simple formulae generate and describe linear number sequences express missing number problems algebraically find pairs of numbers that satisfy an equation with two unknowns

								enumerate possibilities of combinations of two variables solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts solve problems involving the calculation/use of percentages for comparison solve problems involving similar shapes where the scale factor is known or can be found solve problems involving unequal sharing and grouping using knowledge of fractions and multiples
Measurement		Compare size, saying which object is bigger/smaller, longer/shorter, taller/shorter, heavier/lighter Compare capacity saying which has the most/more/least/less					use all four operations to solve problems involving measure [for example, length, mass, volume, money] using decimal notation, including scaling	solve problems involving the calculation and conversion of units of measure, using decimal notation up to 3dp. where appropriate
Money				solve simple problems in a practical context involving addition and subtraction of money of the same unit, including giving change			use all four operations to solve problems involving measure [for example, money]	

Time	Begin to describe a sequence of events, real or fictional, using words such as 'first', 'then					solve problems involving converting from hours to minutes; minutes to seconds; years to months; weeks to days	solve problems involving converting between units of time	
Geometry	Notice and correct an error in a repeating pattern	Compare characteristics of 2D shapes circle, triangle, square, rectangle Compose and decompose shapes to recognise a shape can have other shapes within it						
Statistics				ask and answer simple questions by counting the number of objects in each category and sorting the categories by quantity ask and answer questions about totalling and comparing categorical data	solve one-step and twostep questions [for example, 'How many more?' and 'How many fewer?'] using information presented in scaled bar charts and pictograms and tables	solve comparison, sum and difference problems using information presented in bar charts, pictograms, tables and other graphs	solve comparison, sum and difference problems using information presented in a line graph	calculate and interpret the mean as an average

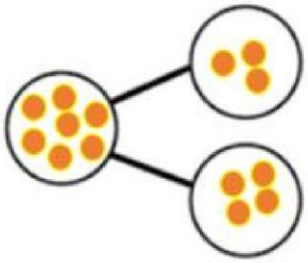
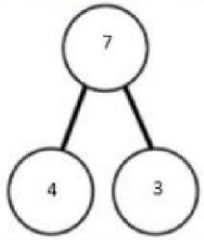
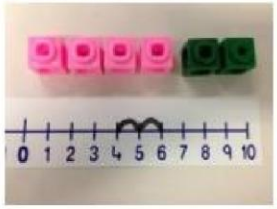
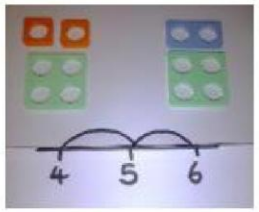
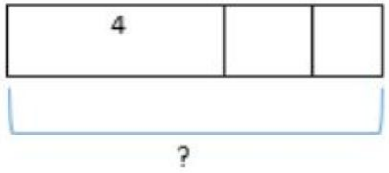
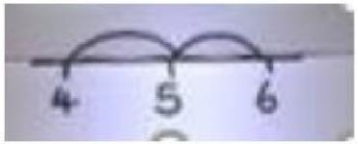
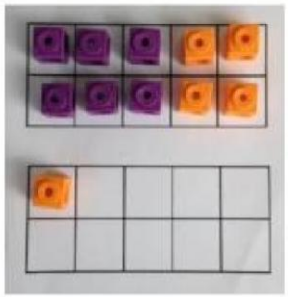
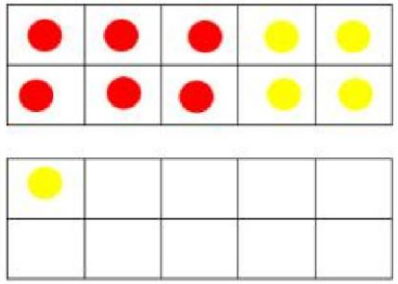
Calculations Overview

	EYFS/Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Addition	Combining two parts to make a whole: part whole model. Starting at the bigger number and counting on- using cubes. Regrouping to make 10 using ten frame.	Adding three single digits. Use of base 10 to combine two numbers.	Column method- regrouping. Using place value counters (up to 3 digits).	Column method- regrouping. (up to 4 digits)	Column method- regrouping. Use of place value counters for adding decimals.	Column method- regrouping. Abstract methods. Place value counters to be used for adding decimal numbers.
Subtraction	Taking away ones Counting back Find the difference Part whole model Make 10 using the ten frame	Counting back Find the difference Part whole model Make 10 Use of base 10	Column method with regrouping. (up to 3 digits using place value counters)	Column method with regrouping. (up to 4 digits)	Column method with regrouping. Abstract for whole numbers. Start with place value counters for decimals- with the same amount of decimal places.	Column method with regrouping. Abstract methods. Place value counters for decimals- with different amounts of decimal places.

Multiplication	Recognising and making equal groups. Doubling Counting in multiples Use cubes, Numicon and other objects in the classroom	Arrays- showing commutative multiplication	Arrays 2d × 1d using base 10	Column multiplication- introduced with place value counters. (2 and 3 digit multiplied by 1 digit)	Column multiplication Abstract only but might need a repeat of year 4 first(up to 4 digit numbers multiplied by 1 or 2 digits)	Column multiplication Abstract methods (multi-digit up to 4 digits by a 2 digit number)
Division	Sharing objects into groups Division as grouping e.g. I have 12 sweets and put them in groups of 3, how many groups? Use cubes and draw round 3 cubes at a time.	Division as grouping Division within arrays- linking to multiplication Repeated subtraction	Division with a remainder-using lollipop sticks, times tables facts and repeated subtraction. 2d divided by 1d using base 10 or place value counters	Division with a remainder Short division (up to 3 digits by 1 digit- concrete and pictorial)	Short division (up to 4 digits by a 1 digit number including remainders)	Short division Long division with place value counters (up to 4 digits by a 2 digit number) Children should exchange into the tenths and hundredths column too

Addition

Key language which should be used: sum, total, parts and wholes, plus, add, altogether, more than, 'is equal to' 'is the same as'

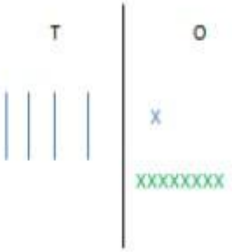
Concrete	Pictorial	Abstract
Combining two parts to make a whole (use other resources too e.g. eggs, shells, teddy bears etc)  		4 + 3 = 7 (four is a part, 3 is a part and the whole is seven) 
Counting on using number lines by using cubes or numicon  	A bar model which encourages the children to count on 	The abstract number line: What is 2 more than 4? What is the sum of 4 and 4? What's the total of 4 and 2? 4 + 2 
Regrouping to make 10 by using ten frames and counters/cubes or using numicon: 6 + 5  	Children to draw the ten frame and counters/cubes 	Children to develop an understanding of equality e.g 6 + ☐ = 11 and 6 + 5 = 5 + ☐ 6 + 5 = ☐ + 4

TO + O using base 10. Continue to develop understanding of partitioning and place value

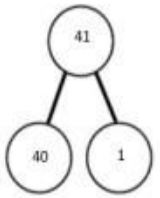
41 + 8



Children to represent the concrete using a particular symbol e.g. lines for tens and dot/crosses for ones.

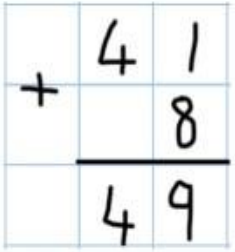


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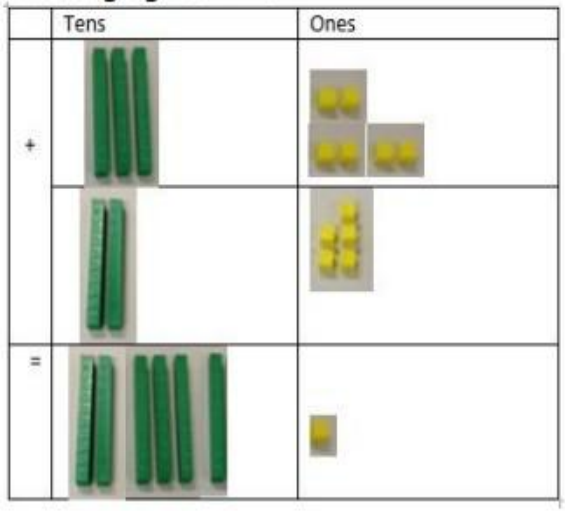


1 + 8 = 9

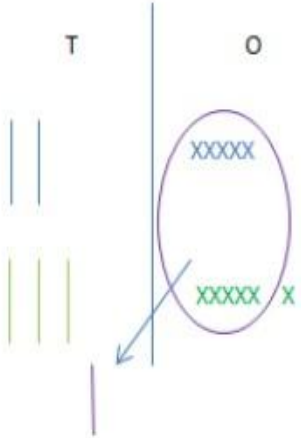
40 + 9 = 49

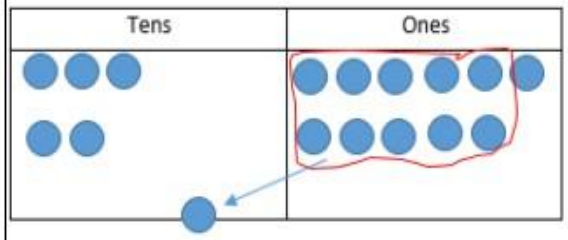


TO + TO using base 10. Continue to develop understanding of partitioning and place value and use this to support addition. Begin with no exchanging. 36 + 25

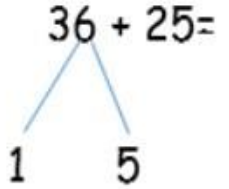


This could be done one of two ways:





Looking for ways to make 10

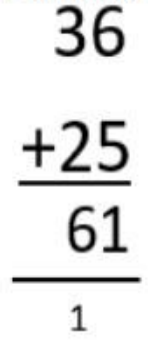


30 + 20 = 50

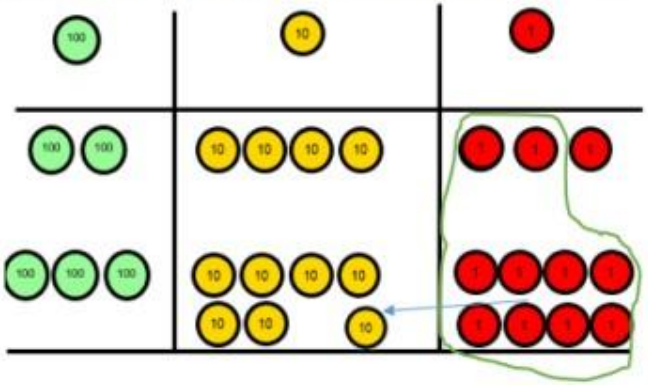
5 + 5 = 10

50 + 10 + 1 = 61

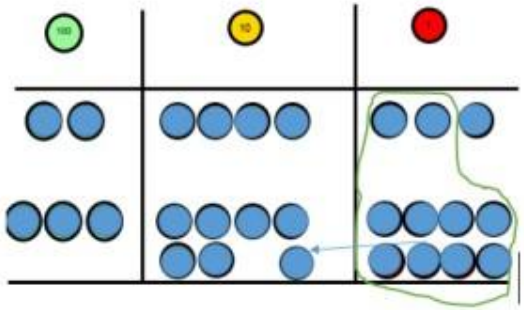
Formal method:



Use of place value counters to add HTO + TO, HTO + HTO etc. once the children have had practice with this, they should be able to apply it to larger numbers and the abstract



Chidren to represent the counters e.g. like the image below

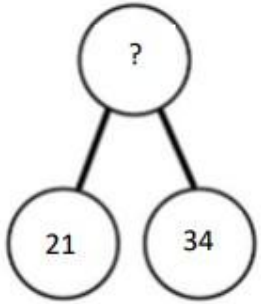


If the children are completing a word problem, draw a bar model to represent what it's asking them to do

?	
243	368

$$\begin{array}{r} 243 \\ +368 \\ \hline 611 \\ \hline 1\ 1 \end{array}$$

Fluency variation, different ways to ask children to solve 21+34:





Sam saved £21 one week and £34 another. How much did he save in total?

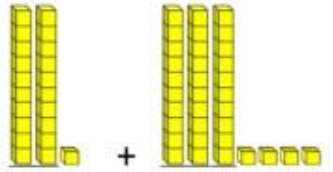
21+34=55. Prove it! (reasoning but the children need to be fluent in representing this)

$$\begin{array}{r} 21 \\ +34 \\ \hline \\ \hline \end{array}$$

21 + 34 =

$\square$ = 21 + 34

What's the sum of twenty one and thirty four?

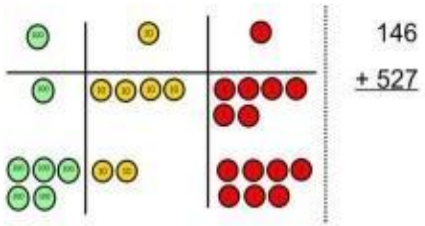


Always use missing digit problems too:

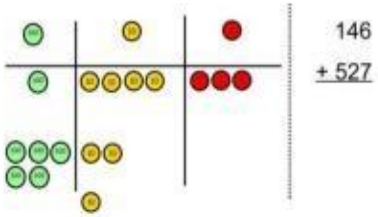
Tens	Ones
	?
?	4

Column method- regrouping

Make both numbers on a place value grid.



Add up the units and exchange 10 ones for one 10.

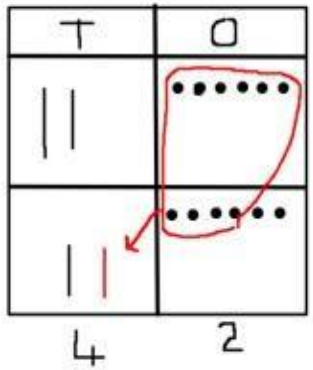


Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added.

This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100.

As children move on to decimals, money and decimal place value counters can be used to support learning.

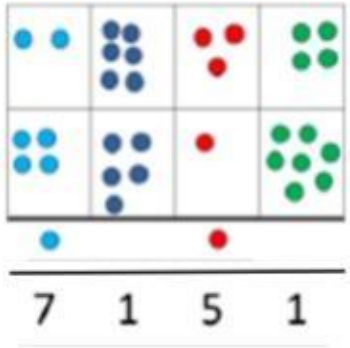
To regroup using the method of drawing base ten, children start by drawing the number sentence in a tens and ones table...



e.g. 26+12=

Children would then start by counting the ones. If they reach 10, they then circle the ten ones and draw an additional ten in the tens column. Children then count the remaining ones – write the number below and count the total tens and write the number below.

Children can draw a pictorial representation of the columns and place value counters to further support their learning and understanding.



Start by partitioning the numbers before moving on to clearly show the exchange below the addition.

$$\begin{array}{rcrl} 20 & + & 5 & \\ 40 & + & 8 & \\ \hline 60 & + & 13 & = 73 \end{array}$$

$$\begin{array}{r} 25 \\ +\,48 \\ \hline {\scriptstyle 1\ \ 1} \\ \hline 73 \end{array}$$

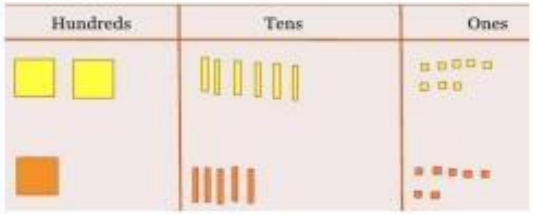
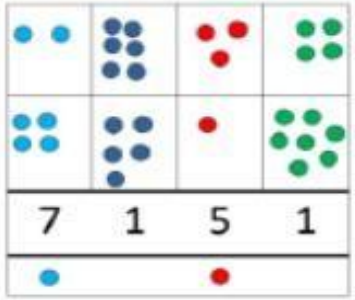
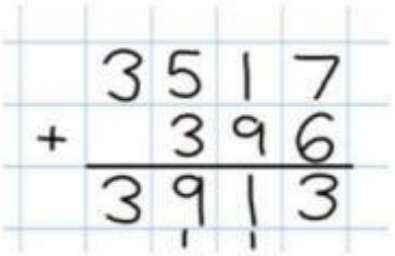
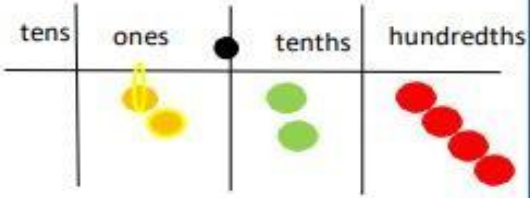
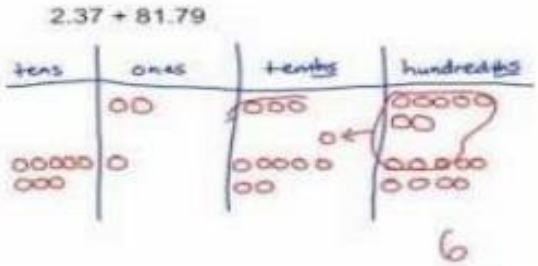
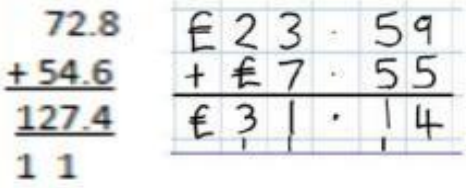
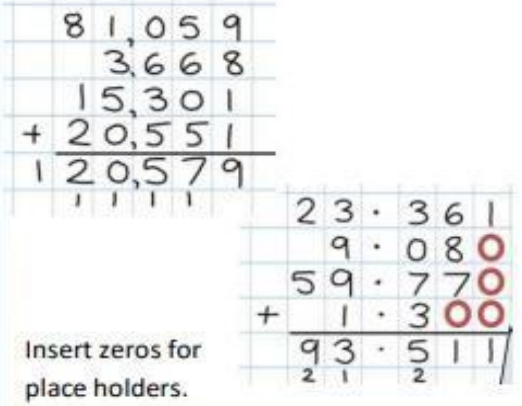
Once children have grasped the exchange above and fully understand the importance of place value. Addition transfers to a simplified version of column addition.

As the children move on, introduce decimals with the same number of decimal places and different. Money can be used here.

$$\begin{array}{r} 72.8 \\ +\,54.6 \\ \hline {\scriptstyle 1\ \ 1\ \ \ 1} \\ \hline 127.4 \end{array}$$

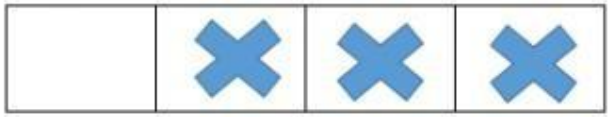
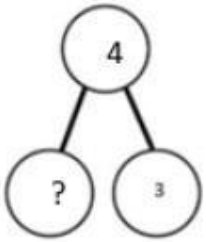
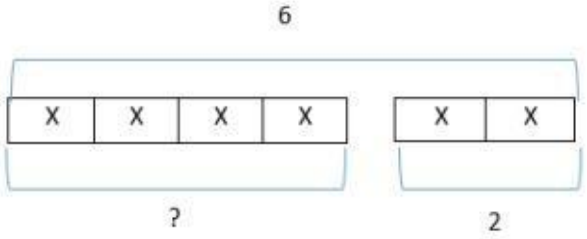
$$\begin{array}{rr} & £23.59 \\ + & £\ \ 7.55 \\ \hline & £31.14 \\ & {\scriptstyle 1\ \ 1\ \ \ 1} \end{array}$$

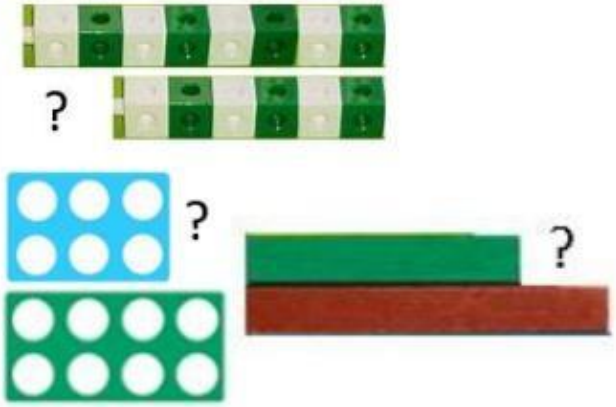
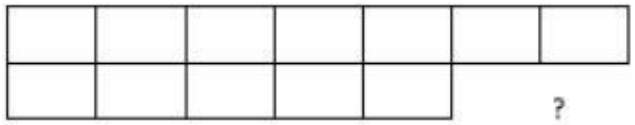
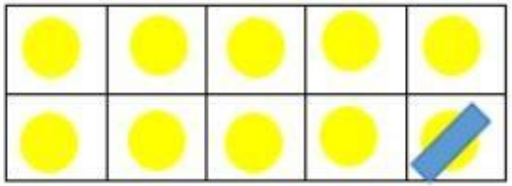
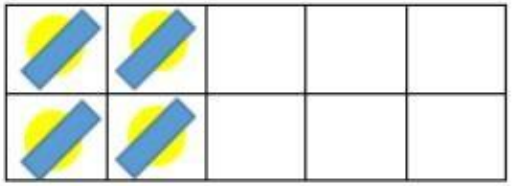
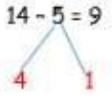
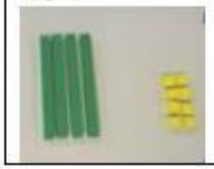
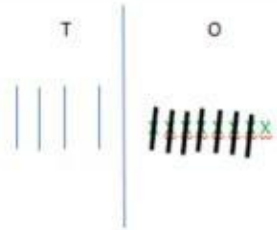
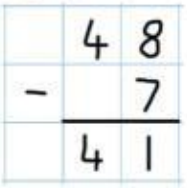
$$\begin{array}{rr} & 23.361 \\ & 9.080 \\ & 59.770 \\ + & 1.300 \\ \hline & 93.511 \\ & {\scriptstyle 2\ \ 1\ \ \ 2} \end{array}$$

Objective & Strategy	Concrete	Pictorial	Abstract
Add numbers with up to 4 digits	Children continue to use dienes or pv counters to add, exchanging ten ones for a ten and ten tens for a hundred and ten hundreds for a thousand. 	 Draw representations using place value grid.	 Continue from previous work to transfer hundreds as well as tens.
Add numbers with more than 4 digits. Add decimals with 2 decimal places, including money.	 Introduce decimal place value counters and model exchange for addition.		Relate to money and measures. 
Add several numbers of increasing complexity Including adding money, measure and decimals with different numbers of decimal points.	As above		 Insert zeros for place holders.

Subtraction

Key language which should be used: take away, less than, the difference, subtract, minus, fewer, decrease, '7 take away 3, the difference is four'

Concrete	Pictorial	Abstract
Physically taking away and removing objects from a whole (use various objects too) rather than crossing out- children will physically remove the objects 4 – 3 = 1 	Children to draw the concrete resources they are using and cross out.  Use of the bar model: 	**4- 3 =** **□ = 4 - 3**  
Counting back (using number lines or number tracks)  	Children to represent what they see pictorially e.g.  	

Finding the difference (using cubes, numicon or Cuisenaire rods, other objects can also be used) 	Children to draw the cubes/other concrete objects which they have used XXXXXXXX XXXXXX Use of the bar model 	Find the difference between 8 and 6. 8 – 6, the difference is ? Children to also explore why 9 - 7 = 8 – 6 (the difference, of each digit, has changed by 1 do the difference is the same- this will help when solving 10000-9987)
Making 10 (using numicon or ten frames) 14 – 5  Children could also do this by subtracting a 5 from the 10. 	Children to present the ten frame pictorially  	14 – 5 = 9 You also want children to see related facts e.g. 15 – 9 = 5 Children to represent how they have solved it e.g.  14 is made up of 5, 5 and 4 so I can subtract one 5 to be left with 4 and 5  5 is made up of 4 and 1 so I can subtract 4 to make 10 and then 1 to get to 9
Column method (using base 10) 48-7 		48 – 7 = 

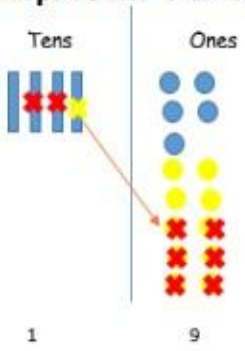
Column method (using base 10 and having to exchange)

45-26

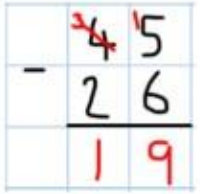


1) Start by partitioning 45
2) Exchange one ten for ten more ones
3) Subtract the ones, then the tens.

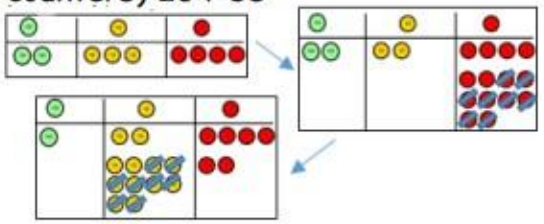
Represent the base 10 pictorially



It's crucial that the children understand that when they have exchanged the 10 they still have 45. 45 = 30 + 15

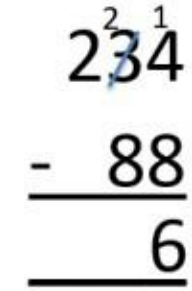


Column method (using place value counters) 234-88

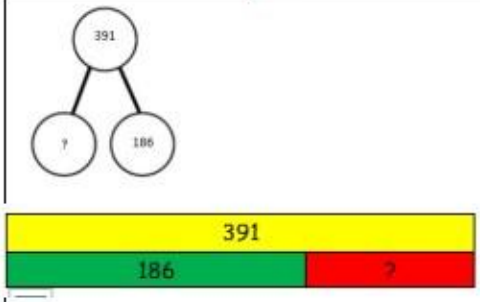


Once the children have had practice with the concrete, they should be able to apply it to any subtraction.

Like the other pictorial representations, children to represent the counters.



Fluency variation, different ways to ask children to solve 391-186:



Raj spent £391, Timmy spent £186. How much more did Raj spend?

I had 391 metres to run. After 186 I stopped. How many metres do I have left to run?

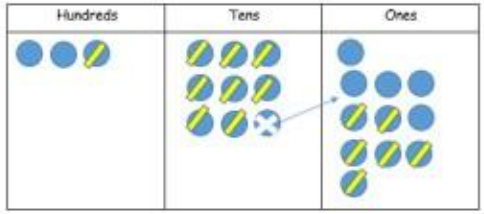
391 - 186

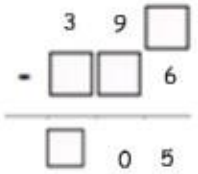
□ = 391 - 186

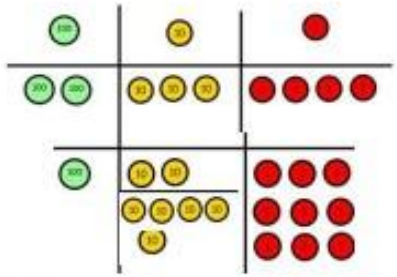
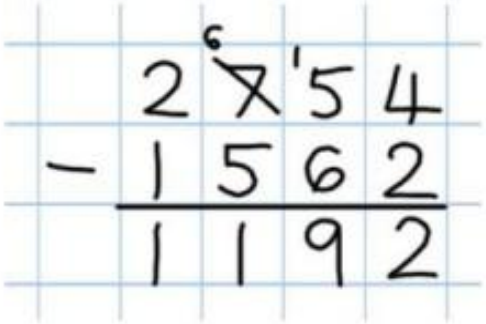
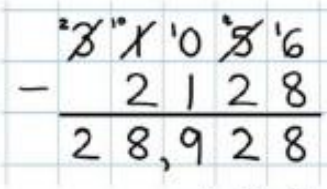
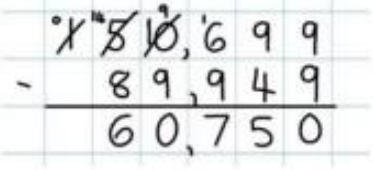
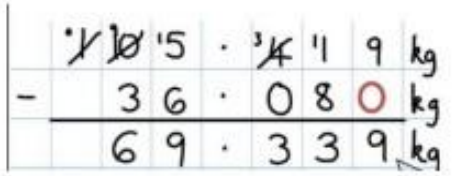
391
-186
____

Find the difference ebtween 391 and 186
Subtract 186 from 391.
What is 186 less than 391?

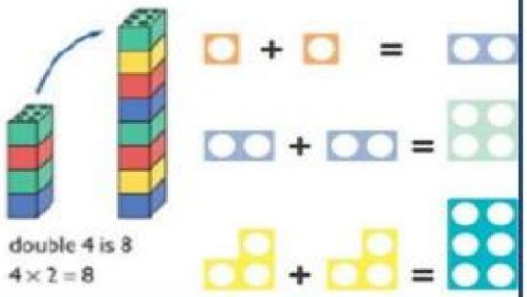
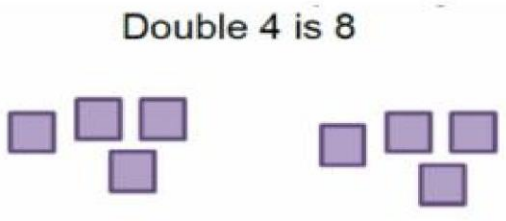
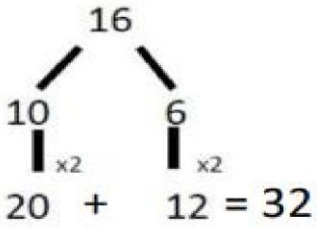
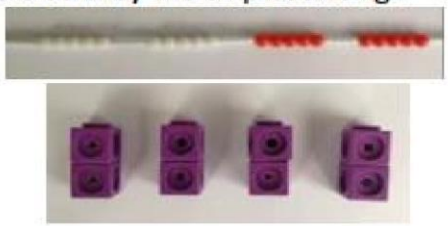
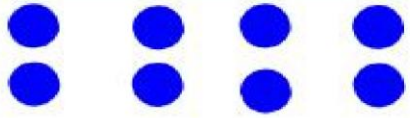
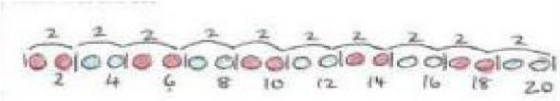
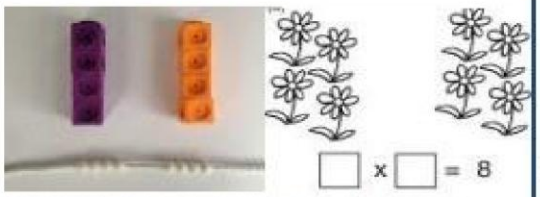
What's the calculation? What's the answer?

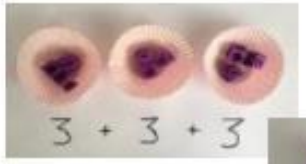
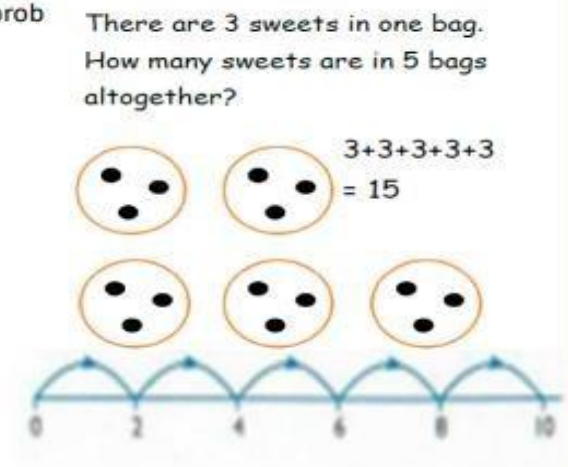
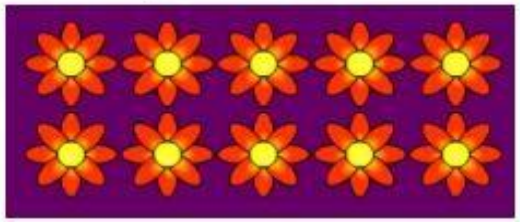
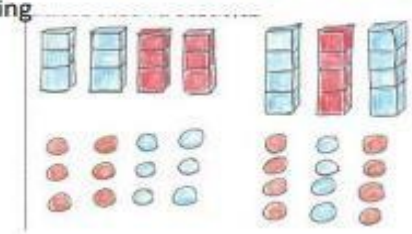


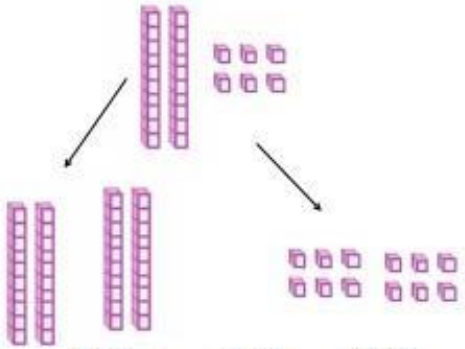
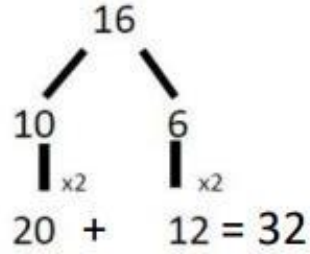
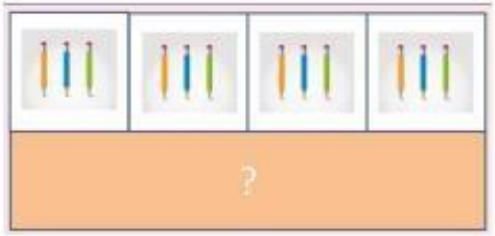
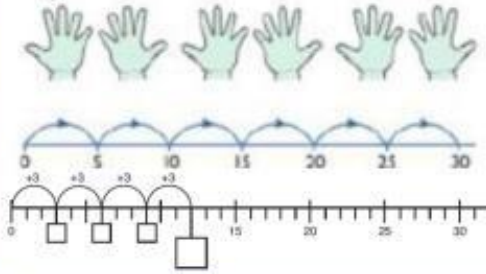
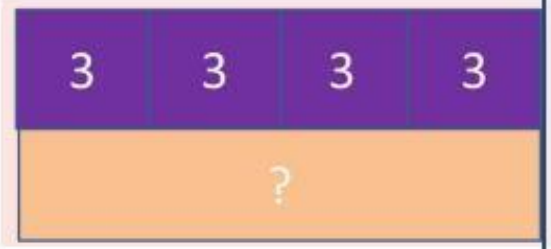


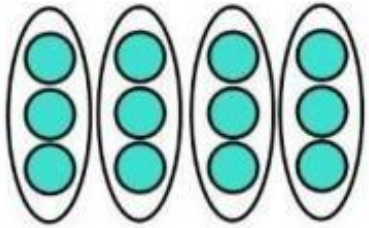
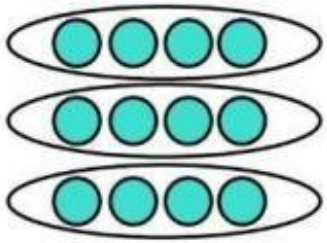
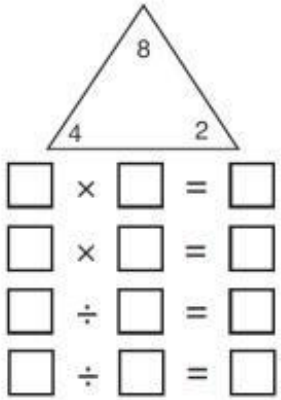
Objective & Strategy	Concrete	Pictorial	Abstract
Subtracting tens and ones Subtract with up to 4 digits. *Introduce decimal subtraction through context of money*	234 - 179  Model process of exchange using Numicon, base ten and then move to Place Value counters.	Children to draw place value counters and show their exchange	 Use the phrases transfer and exchange
Subtract with at least 4 dig- its, including money and measures. *Subtract with decimal values, including mixtures of integers and decimals and aligning the decimal*	As above.	Children to draw place value counters and show their exchange.	 Use zeros for place-holders. 
Subtract with increasingly large and more complex numbers and decimal values.	By this stage, standard written methods should be secure.		 

Multiplication

Objective & Strategy	Concrete	Pictorial	Abstract
Doubling	Use practical activities using manipultives (including cubes and Numicon) to demonstrate doubling  	Draw pictures to show how to double numbers  	Partition a number and then double each part before recombining it back together. 
Counting in multi-ples	Count the groups as children are skip counting, children may use their fingers as they are skip counting. 	 Children make representations to show counting in multiples. 	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25 , 30
Making equal groups and counting the total	 Use manipulatives to create equal groups.	Draw  to show 2 x 3 = 6 Draw and make representations	2 x 4 = 8

Objective & Strategy	Concrete	Pictorial	Abstract
Repeated addition	   Use different objects to add equal groups	Use pictorial including number lines to solve prob 	Write addition sentences to describe objects and pictures. 
Understanding arrays	Use objects laid out in arrays to find the answers to 2 lots 5, 3 lots of 2 etc. 	Draw representations of arrays to show understanding 	3 x 2 = 6 2 x 5 = 10

Objective & Strategy	Concrete	Pictorial	Abstract
Doubling	Model doubling using dienes and PV counters.  40 + 12 = 52	Draw pictures and representations to show how to double numbers	Partition a number and then double each part before recombining it back together.  20 + 12 = 32
Counting in multiples of 2, 3, 4, 5, 10 from 0 (repeated addition)	Count the groups as children are skip counting, children may use their fingers as they are skip counting. Use bar models.  5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 40   	Number lines, counting sticks and bar models should be used to show representation of counting in multiples.   	Count in multiples of a number aloud. Write sequences with multiples of numbers. 0, 2, 4, 6, 8, 10 0, 3, 6, 9, 12, 15 0, 5, 10, 15, 20, 25 , 30 $4 \times 3 = \square$

Objective & Strategy	Concrete	Pictorial	Abstract
Multiplication is commutative	Create arrays using counters and cubes and Numicon.    Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.  	Use representations of arrays to show different calculations and explore commutativity.  	$12 = 3 \times 4$ $12 = 4 \times 3$ Use an array to write multiplication sentences and reinforce repeated addition.  $5 + 5 + 5 = 15$ $3 + 3 + 3 + 3 + 3 = 15$ $5 \times 3 = 15$ $3 \times 5 = 15$
Using the Inverse *This should be taught alongside division, so pupils learn how they work alongside each other.*			2 x 4 = 8 4 x 2 = 8 8 ÷ 2 = 4 8 ÷ 4 = 2 8 = 2 x 4 8 = 4 x 2 2 = 8 ÷ 4 4 = 8÷ 2 Show all 8 related fact family sentences.

Skill: Multiply 2-digit numbers by 1-digit numbers

Year: 3/4



	H	T	O	
		3	4	
×			5	
		2	0	(5 × 4)
+	1	5	0	(5 × 30)
	1	7	0	

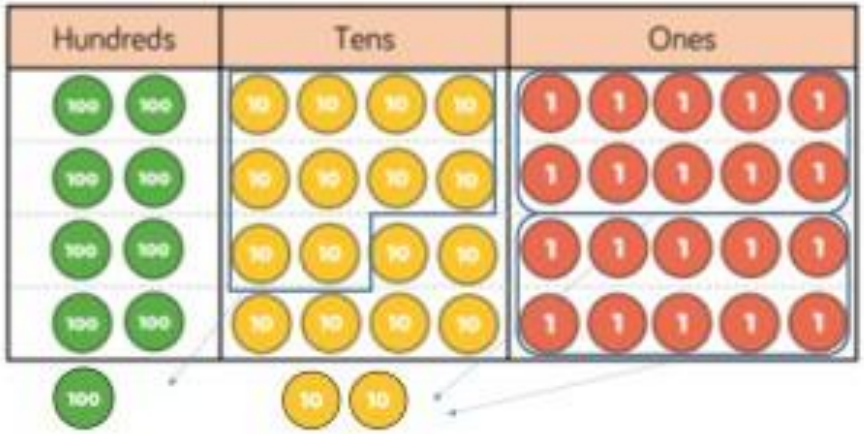
$$34 \times 5 = 170$$

	H	T	O	
		3	4	
×			5	
	1	7	0	
	1	2		



Informal methods and the expanded method are used in Year 3 before moving on to the short multiplication method in Year 4.

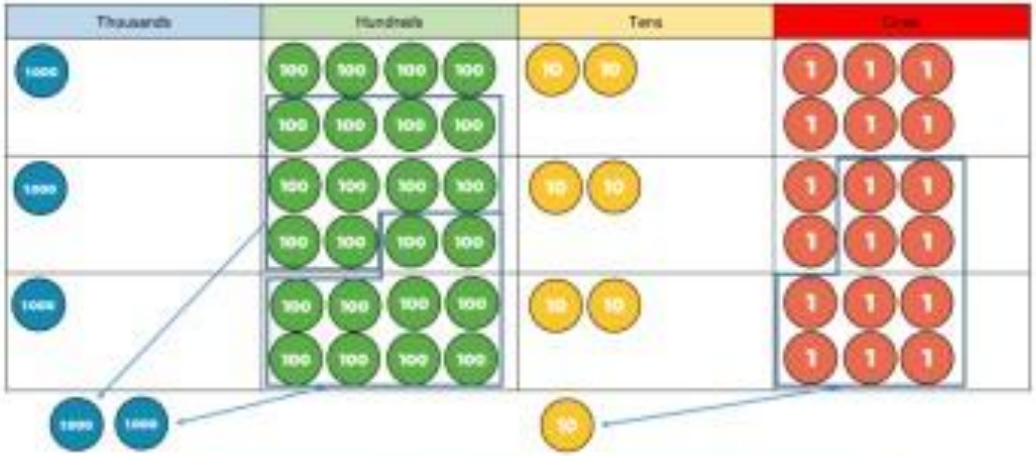
Place value counters should be used to support the understanding of the method rather than supporting the multiplication, as children should use times table knowledge.

Skill: Multiply 3-digit numbers by 1-digit numbers	Year: 4
 $245 \times 4 = 980$ 	When moving to 3-digit by 1-digit multiplication, encourage children to move towards the short, formal written method. Base 10 and place value counters continue to support the understanding of the written method. Limit the number of exchanges needed in the questions and move children away from resources when multiplying larger numbers.

	H	T	O
	2	4	5
×			4
	9	8	0
	1	2	

Skill: Multiply 4-digit numbers by 1-digit numbers

Year: 5

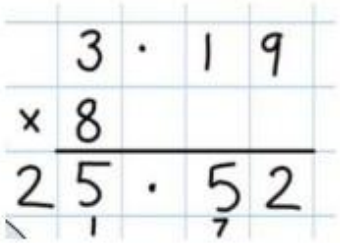


$$1{,}826 \times 3 = 5{,}478$$

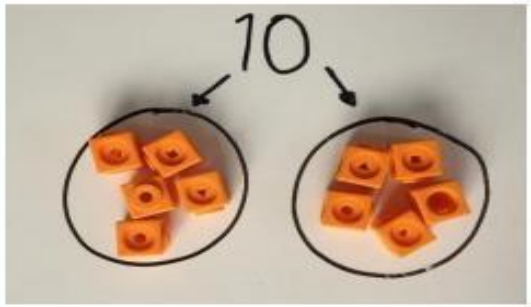
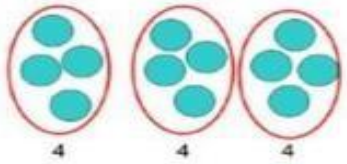
	Th	H	T	O
	1	8	2	6
×				3
	5	4	7	8
	2		1	

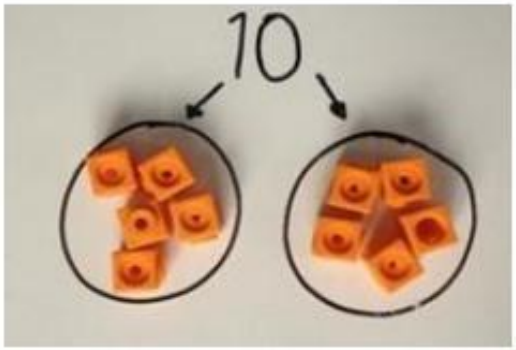
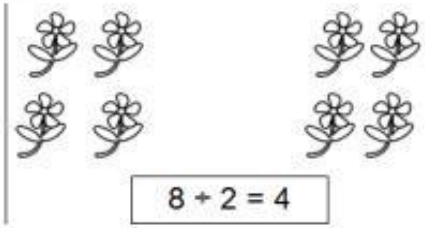
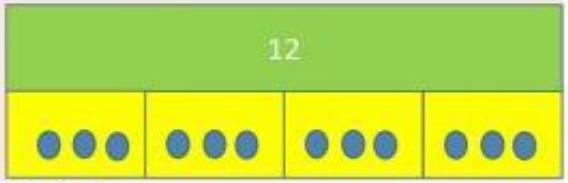
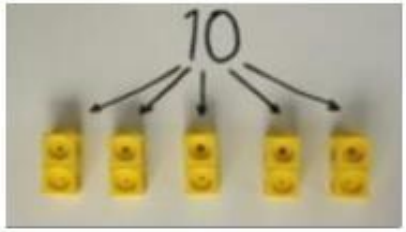
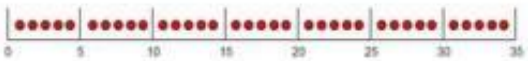
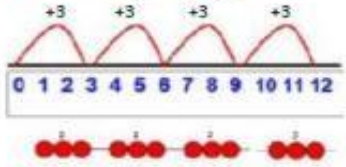
When multiplying 4-digit numbers, place value counters are the best manipulative to use to support children in their understanding of the formal written method.
If children are multiplying larger numbers and struggling with their times tables, encourage the use of multiplication grids so children can focus on the use of the written method.

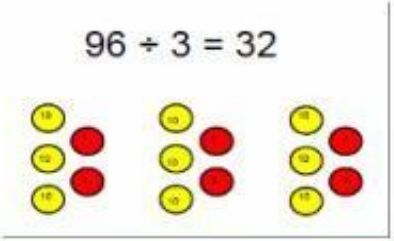
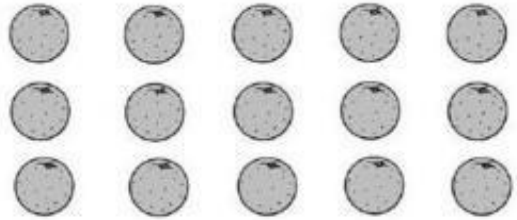
Skill: Multiply 4-digit numbers by 2-digit numbers	Year: 5/6
 **2,739 × 28 = 76,692**	When multiplying 4-digits by 2-digits, children should be confident in using the formal written method. If they are still struggling with times tables, provide multiplication grids to support when they are focusing on the use of the method. Consider where exchanged digits are placed and make sure this is consistent.

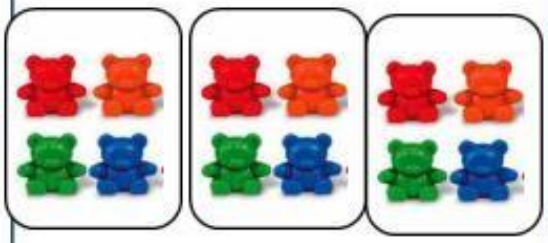
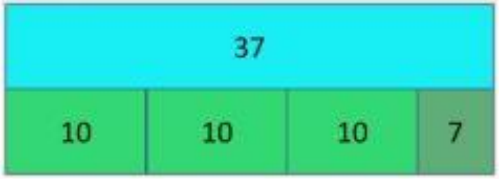
Objective & Strategy	Concrete	Pictorial	Abstract
Multiplying decimals up to 2 decimal places by a single digit.	At this stage, grid method may also be reintroduced, following earlier pattern for whole numbers.		Remind children that the single digit belongs in the units column. Line up the decimal points in the question and the answer. 

Division

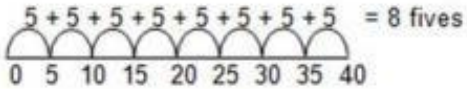
Objective & Strategy	Concrete	Pictorial	Abstract
Division as sharing *Use Gordon ITPs for modelling*	   I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities.  8 shared between 2 is 4 Sharing:  12 shared between 3 is 4	12 shared between 3 is 4

Objective & Strategy	Concrete	Pictorial	Abstract
Division as sharing	 I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities.  8 ÷ 2 = 4 Children use bar modelling to show and support understanding.  12 ÷ 4 = 3	$12 \div 3 = 4$
Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.  	Use number lines for grouping  12 ÷ 3 = 4 Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.  20 ÷ 5 = ? 5 x ? = 20	28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group?

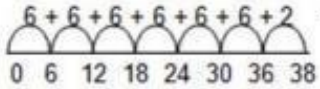
Objective & Strategy	Concrete	Pictorial	Abstract
Division as grouping	Use cubes, counters, objects or place value counters to aid understanding.  24 divided into groups of 6 = 4 	Continue to use bar modelling to aid solving division problems.  20 ÷ 5 = ? 5 x ? = 20	How many groups of 6 in 24? 24 ÷ 6 = 4
Division with arrays	 Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	Draw an array and use lines to split the array into groups to make multiplication and division sentences 	Find the inverse of multiplication and division sentences by creating eight linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7 28 = 7 x 4 28 = 4 x 7 4 = 28 ÷ 7 7 = 28 ÷ 4

Objective & Strategy	Concrete	Pictorial	Abstract
Division with remainders.	14 ÷ 3 = Divide objects between groups and see how much is left over  	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.  Draw dots and group them to divide an amount and clearly show a remainder.  Use bar models to show division with remainders.  	Complete written divisions and show the remainder using r. 

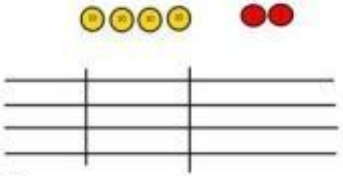
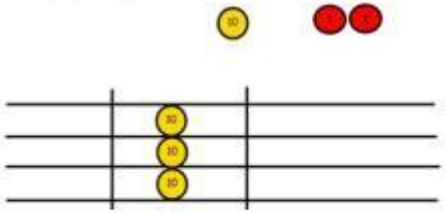
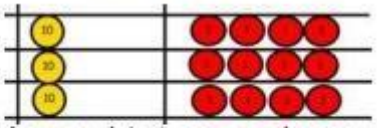
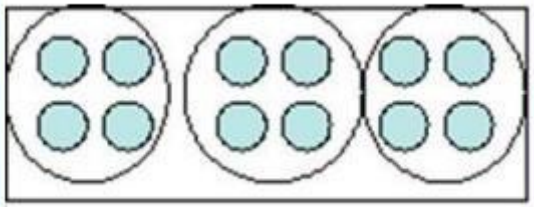
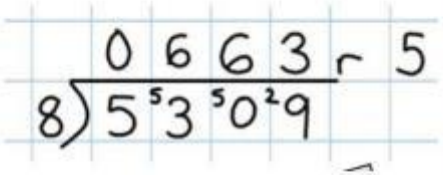
Example without remainder:
40 ÷ 5
Ask "How many 5s in 40?"



Example with remainder:
38 ÷ 6



For larger numbers, when it becomes inefficient to count in single multiples, bigger jumps can be recorded using known facts.

Objective & Strategy	Concrete	Pictorial	Abstract
Divide at least 3 digit numbers by 1 digit. Short Division	96 ÷ 3 Tens Units 3 2 3  Use place value counters to divide using the bus stop method alongside  Calculations 42 ÷ 3 42 ÷ 3= Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.  We exchange this ten for ten ones and then share the ones equally among the groups.  We look how much in 1 group so the answer is 14.	Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.  Encourage them to move towards counting in multiples to divide more efficiently.	Begin with divisions that divide equally with no remainder. 2 1 8 3 4 8 7 2 Move onto divisions with a remainder. 8 6 r 2 3 5 4 3 2 Finally move into decimal places to divide the total accurately. 1 4 . 6 16 21 3 5 5 1 1 . 0  0 6 6 3 r 5 8 5 5 3 0 2 9

Skill: Divide multi digits by 2-digits (short division)	Year: 6

		0	3	6
	12	4	$^{4}3$	$^{7}2$

$432 \div 12 = 36$

$7{,}335 \div 15 = 489$

	0	4	8	9
15	7	$^{7}3$	$^{13}3$	$^{13}5$

15	30	45	60	75	90	105	120	135	150

When children begin to divide up to 4-digits by 2-digits, written methods become the most accurate as concrete and pictorial representations become less effective. Children can write out multiples to support their calculations with larger remainders. Children will also solve problems with remainders where the quotient can be rounded as appropriate.

Skill: Divide multi-digits by 2-digits (long division)	Year: 6

		0	3	6	
1	2	4	3	2	(×30)
	–	3	6	0	
			7	2	(×6)
	–		7	2	
				0	

$12 \times 1 = 12$
$12 \times 2 = 24$
$12 \times 3 = 36$
$12 \times 4 = 48$
$12 \times 5 = 60$
$12 \times 6 = 72$
$12 \times 7 = 84$
$12 \times 8 = 96$
$12 \times 7 = 108$
$12 \times 10 = 120$

$432 \div 12 = 36$

$7{,}335 \div 15 = 489$

	0	4	8	9	
15	7	3	3	5	
–	6	0	0	0	(×400
	1	3	3	5	
–	1	2	0	0	(×80)
		1	3	5	
–		1	3	5	(×9)
				0	

$1 \times 15 = 15$
$2 \times 15 = 30$
$3 \times 15 = 45$
$4 \times 15 = 60$
$5 \times 15 = 75$
$10 \times 15 = 150$

Children can also divide by 2-digit numbers using long division.

Children can write out multiples to support their calculations with larger remainders.

Children will also solve problems with remainders where the quotient can be rounded as appropriate.

Skill: Divide multi digits by 2-digits (long division)	Year: 6

$372 \div 15 = 24\ r12$

			2	4	r	1	2
1	5	3	7	2			
	–	3	0	0			
			7	2			
	–		6	0			
			1	2			

$1 \times 15 = 15$
$2 \times 15 = 30$
$3 \times 15 = 45$
$4 \times 15 = 60$
$5 \times 15 = 75$
$10 \times 15 = 150$

When a remainder is left at the end of a calculation, children can either leave it as a remainder or convert it to a fraction.
This will depend on the context of the question.

			2	4	$\frac{4}{5}$
1	5	3	7	2	
	–	3	0	0	
			7	2	
	–		6	0	
			1	2	

$372 \div 15 = 24\frac{4}{5}$

Children can also answer questions where the quotient needs to be rounded according to the context.

Glossary

Addend - A number to be added to another.

Aggregation - combining two or more quantities or measures to find a total.

Augmentation - increasing a quantity or measure by another quantity.

Commutative – numbers can be added in any order.

Complement – in addition, a number and its complement make a total e.g. 300 is the complement to 700 to make 1,000

Difference – the numerical difference between two numbers is found by comparing the quantity in each group.

Exchange – Change a number or expression for another of an equal value.

Minuend – A quantity or number from which another is subtracted.

Partitioning – Splitting a number into its component parts.

Reduction – Subtraction as take away.

Subitise – Instantly recognise the number of objects in a small group without needing to count.

Subtrahend - A number to be subtracted from another.

Sum - The result of an addition.

Total – The aggregate or the sum found by addition.

Array – An ordered collection of counters, cubes or other item in rows and columns.

Commutative – Numbers can be multiplied in any order.

Dividend – In division, the number that is divided.

Divisor – In division, the number by which another is divided.

Exchange – Change a number or expression for another of an equal value.

Factor – A number that multiplies with another to make a product.

Multiplicand – In multiplication, a number to be multiplied by another.

Partitioning – Splitting a number into its component parts.

Product – The result of multiplying one number by another.

Quotient – The result of a division

Remainder – The amount left over after a division when the divisor is not a factor of the dividend.

Scaling – Enlarging or reducing a number by a given amount, called the scale factor